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Infinite Series Expressions for the Values of Some Fractional Analytic Functions

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Abstract: **In this paper, we find the infinite series expressions for the values of some fractional analytic functions. Jumarie's modified Riemann-Liouville (R-L) fractional calculus and a new multiplication of fractional analytic functions play important roles in this article. In fact, our results are generalizations of classical calculus results.**

Keywords: **Infinite series expressions, fractional analytic functions, Jumarie's modified R-L fractional calculus, new multiplication.**

I. INTRODUCTION

In 1695, the concept of fractional derivative first appeared in a famous letter between L'Hospital and Leibniz. Many great mathematicians have further developed this field. We can mention Euler, Lagrange, Laplace, Fourier, Abel, Liouville, Riemann, Hardy, Littlewood, and Weyl. Fractional calculus has important applications in various fields such as physics, mechanics, electrical engineering, biology, economics, viscoelasticity, control theory, and so on [1-11].

However, the definition of fractional derivative is not unique. Common definitions include Riemann-Liouville (R-L) fractional calculus, Caputo fractional calculus, Grunwald-Letnikov (G-L) fractional calculus, and Jumarie's modified R-L fractional calculus [12-16]. Because Jumarie type of R-L fractional calculus helps to avoid non-zero fractional derivative of constant function, it is easier to use this definition to connect fractional calculus with traditional calculus.

In this paper, we obtain the infinite series expressions for the values of some fractional analytic functions. Jumarie's modified R-L fractional calculus and a new multiplication of fractional analytic functions play important roles in this paper. Some examples are given to illustrate our results. In fact, our results are generalizations of ordinary calculus results.

II. PRELIMINARIES

Firstly, we introduce the fractional calculus used in this paper and its properties.

Definition 2.1 ([17])**:** Let $0 < \alpha \leq 1$, and x_0 be a real number. The Jumarie type of Riemann-Liouville (R-L) α-fractional derivative is defined by

$$\left({}_{x_0}D_x^{\alpha}\right)[f(x)] = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_{x_0}^{x}\frac{f(t)-f(x_0)}{(x-t)^{\alpha}}dt\ . \quad (1)$$

And the Jumarie type of R-L α-fractional integral is defined by

$$\left({}_{x_0}I_x^{\alpha}\right)[f(x)] = \frac{1}{\Gamma(\alpha)}\int_{x_0}^{x}\frac{f(t)}{(x-t)^{1-\alpha}}dt\ , \quad (2)$$

where $\Gamma(\ \ )$ is the gamma function.

Proposition 2.2 ([18])**:** *If* α, β, x_0, c *are real numbers and* $\beta \geq \alpha > 0$, *then*

$$\left({}_{x_0}D_x^{\alpha}\right)\left[(x-x_0)^{\beta}\right] = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}(x-x_0)^{\beta-\alpha}, \quad (3)$$

and

$$\left({}_{x_0}D_x^{\alpha}\right)[c] = 0. \tag{4}$$

Next, the definition of fractional analytic function is introduced.

Definition 2.3 ([19])**:** Suppose that x, x_0, and a_n are real numbers for all n, $x_0 \in (a,b)$, and $0 < \alpha \leq 1$. If the function $f_\alpha: [a,b] \to R$ can be expressed as an α-fractional power series, that is, $f_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{a_n}{\Gamma(n\alpha+1)}(x - x_0)^{n\alpha}$ on some open interval containing x_0, then we say that $f_\alpha(x^\alpha)$ is α-fractional analytic at x_0. In addition, if $f_\alpha: [a,b] \to R$ is continuous on closed interval $[a,b]$ and it is α-fractional analytic at every point in open interval (a,b), then f_α is called an α-fractional analytic function on $[a,b]$.

In the following, we introduce a new multiplication of fractional analytic functions.

Definition 2.4 ([20])**:** Let $0 < \alpha \leq 1$, and x_0 be a real number. If $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ are two α-fractional analytic functions defined on an interval containing x_0,

$$f_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{a_n}{\Gamma(n\alpha+1)}(x - x_0)^{n\alpha}, \tag{5}$$

$$g_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{b_n}{\Gamma(n\alpha+1)}(x - x_0)^{n\alpha}. \tag{6}$$

Then we define

$$\begin{aligned} & f_\alpha(x^\alpha) \otimes_\alpha g_\alpha(x^\alpha) \\ &= \sum_{n=0}^{\infty} \frac{a_n}{\Gamma(n\alpha+1)}(x - x_0)^{n\alpha} \otimes_\alpha \sum_{n=0}^{\infty} \frac{b_n}{\Gamma(n\alpha+1)}(x - x_0)^{n\alpha} \\ &= \sum_{n=0}^{\infty} \frac{1}{\Gamma(n\alpha+1)}\left(\sum_{m=0}^{n} \binom{n}{m} a_{n-m} b_m\right)(x - x_0)^{n\alpha}. \end{aligned} \tag{7}$$

Equivalently,

$$\begin{aligned} & f_\alpha(x^\alpha) \otimes_\alpha g_\alpha(x^\alpha) \\ &= \sum_{n=0}^{\infty} \frac{a_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x - x_0)^{\alpha}\right)^{\otimes_\alpha n} \otimes_\alpha \sum_{n=0}^{\infty} \frac{b_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x - x_0)^{\alpha}\right)^{\otimes_\alpha n} \\ &= \sum_{n=0}^{\infty} \frac{1}{n!}\left(\sum_{m=0}^{n} \binom{n}{m} a_{n-m} b_m\right)\left(\frac{1}{\Gamma(\alpha+1)}(x - x_0)^{\alpha}\right)^{\otimes_\alpha n}. \end{aligned} \tag{8}$$

Definition 2.5 ([21])**:** If $0 < \alpha \leq 1$, and $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ are two α-fractional analytic functions defined on an interval containing x_0,

$$f_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{a_n}{\Gamma(n\alpha+1)}(x - x_0)^{n\alpha} = \sum_{n=0}^{\infty} \frac{a_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x - x_0)^{\alpha}\right)^{\otimes_\alpha n}, \tag{9}$$

$$g_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{b_n}{\Gamma(n\alpha+1)}(x - x_0)^{n\alpha} = \sum_{n=0}^{\infty} \frac{b_n}{n!}\left(\frac{1}{\Gamma(\alpha+1)}(x - x_0)^{\alpha}\right)^{\otimes_\alpha n}. \tag{10}$$

The compositions of $f_\alpha(x^\alpha)$ and $g_\alpha(x^\alpha)$ are defined by

$$(f_\alpha \circ g_\alpha)(x^\alpha) = f_\alpha\left(g_\alpha(x^\alpha)\right) = \sum_{n=0}^{\infty} \frac{a_n}{n!}\left(g_\alpha(x^\alpha)\right)^{\otimes_\alpha n}, \tag{11}$$

and

$$(g_\alpha \circ f_\alpha)(x^\alpha) = g_\alpha\left(f_\alpha(x^\alpha)\right) = \sum_{n=0}^{\infty} \frac{b_n}{n!}\left(f_\alpha(x^\alpha)\right)^{\otimes_\alpha n}. \tag{12}$$

Definition 2.6 ([22, 23])**:** If $0 < \alpha \leq 1$, and x is a real variable. The α-fractional exponential function is defined by

$$E_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{x^{n\alpha}}{\Gamma(n\alpha+1)} = \sum_{n=0}^{\infty} \frac{1}{n!}\left(\frac{1}{\Gamma(\alpha+1)} x^{\alpha}\right)^{\otimes_\alpha n}. \tag{13}$$

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And the α-fractional logarithmic function $Ln_\alpha(x^\alpha)$ is the inverse function of $E_\alpha(x^\alpha)$. On the other hand, the α-fractional cosine and sine function are defined as follows:

$$cos_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^k x^{2n\alpha}}{\Gamma(2n\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes_\alpha 2n}, \tag{14}$$

and

$$sin_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)\alpha}}{\Gamma((2n+1)\alpha+1)} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes_\alpha (2n+1)}. \tag{15}$$

Definition 2.7: Let $0 < \alpha \le 1$, and $f_\alpha(x^\alpha)$, $g_\alpha(x^\alpha)$ be two α-fractional analytic functions. Then $\left(f_\alpha(x^\alpha)\right)^{\otimes_\alpha n} = f_\alpha(x^\alpha) \otimes_\alpha \cdots \otimes_\alpha f_\alpha(x^\alpha)$ is called the nth power of $f_\alpha(x^\alpha)$. On the other hand, if $f_\alpha(x^\alpha) \otimes_\alpha g_\alpha(x^\alpha) = 1$, then $g_\alpha(x^\alpha)$ is called the $\otimes_\alpha$ reciprocal of $f_\alpha(x^\alpha)$, and is denoted by $\left(f_\alpha(x^\alpha)\right)^{\otimes_\alpha -1}$.

Definition 2.8 ([24])**:** Let $0 < \alpha \le 1$. If $u_\alpha(x^\alpha)$ is a α-fractional analytic function and r is a real number. Then the α-fractional rth power function $u_\alpha(x^\alpha)^{\otimes_\alpha r}$ is defined by

$$u_\alpha(x^\alpha)^{\otimes_\alpha r} = E_\alpha\left(r \cdot Ln_\alpha\left(u_\alpha(x^\alpha)\right)\right). \tag{16}$$

Definition 2.9: The smallest positive real number T_α such that $E_\alpha(iT_\alpha) = 1$, is called the period of $E_\alpha(ix^\alpha)$.

Definition 2.10 (fractional binomial series) ([25])**:** If $0 < \alpha \le 1$, and r is a real number, then the α-fractional binomial series

$$\left(1 + \frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes_\alpha r} = \sum_{n=0}^{\infty} \frac{(r)_n}{n!} \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes_\alpha n\alpha} = \sum_{n=0}^{\infty} \frac{(r)_n}{\Gamma(n\alpha+1)} x^{n\alpha}, \tag{17}$$

where $-1 < \frac{1}{\Gamma(\alpha+1)} x^\alpha < 1$, and $(r)_n = r(r-1)\cdots(r-n+1)$ for any positive integer n, $(r)_0 = 1$.

III. EXAMPLES

In this section, some examples are proposed to illustrate how to find infinite series expressions for the values of fractional analytic functions.

Example 3.1: Let $0 < \alpha \le 1$. Find $E_\alpha(1)$ and $E_\alpha\left(\frac{1}{2}\right)$.

Solution Since $E_\alpha(x^\alpha) = \sum_{n=0}^{\infty} \frac{x^{n\alpha}}{\Gamma(n\alpha+1)}$, it follows that

$$E_\alpha(1) = \sum_{n=0}^{\infty} \frac{[\Gamma(\alpha+1)]^n}{\Gamma(n\alpha+1)}, \tag{18}$$

and

$$E_\alpha\left(\frac{1}{2}\right) = \sum_{n=0}^{\infty} \frac{\left[\frac{1}{2} \cdot \Gamma(\alpha+1)\right]^n}{\Gamma(n\alpha+1)} = \sum_{n=0}^{\infty} \frac{[\Gamma(\alpha+1)]^n}{2^n \cdot \Gamma(n\alpha+1)}. \tag{19}$$

Example 3.2: Let $0 < \alpha \le 1$. Find $Ln_\alpha(2)$.

Solution If $-1 < \frac{1}{\Gamma(\alpha+1)} x^\alpha < 1$, then

$$Ln_\alpha\left(1 + \frac{1}{\Gamma(\alpha+1)} x^\alpha\right)$$

$$= \left({}_0I_x^\alpha\right)\left[\left(1 + \frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes_\alpha -1}\right]$$

$$= \left({}_0I_x^\alpha\right)\left[1 - \frac{1}{\Gamma(\alpha+1)} x^\alpha + \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes_\alpha 2} - \left(\frac{1}{\Gamma(\alpha+1)} x^\alpha\right)^{\otimes_\alpha 3} + \cdots\right]$$

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$$= \left({}_0I_x^{\alpha}\right)[1] - \left({}_0I_x^{\alpha}\right)\left[\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right] + \left({}_0I_x^{\alpha}\right)\left[\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha 2}\right] - \left({}_0I_x^{\alpha}\right)\left[\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha 3}\right] + \cdots$$

$$= \frac{1}{\Gamma(\alpha+1)}x^{\alpha} - \frac{1}{2}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha 2} + \frac{1}{3}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha 3} - \frac{1}{4}\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha 4} + \cdots$$

$$= \frac{1}{\Gamma(\alpha+1)}x^{\alpha} - \frac{1!}{\Gamma(2\alpha+1)}x^{2\alpha} + \frac{2!}{\Gamma(3\alpha+1)}x^{3\alpha} - \frac{3!}{\Gamma(4\alpha+1)}x^{4\alpha} + \cdots$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{n!}{\Gamma((n+1)\alpha+1)}x^{(n+1)\alpha}\ . \tag{20}$$

Therefore,

$$Ln_{\alpha}(2) = \sum_{n=0}^{\infty}(-1)^n \frac{n!\cdot[\Gamma(\alpha+1)]^{n+1}}{\Gamma((n+1)\alpha+1)}\ , \tag{21}$$

if $\sum_{n=0}^{\infty}(-1)^n \frac{n!\cdot[\Gamma(\alpha+1)]^{n+1}}{\Gamma((n+1)\alpha+1)}$ exists.

Example 3.3: Assume that $0 < \alpha \leq 1$. Find $cos_{\alpha}(1)$ and $sin_{\alpha}(1)$.

Solution Since $cos_{\alpha}(x^{\alpha}) = \sum_{n=0}^{\infty} \frac{(-1)^k x^{2n\alpha}}{\Gamma(2n\alpha+1)}$, it follows that

$$cos_{\alpha}(1) = \sum_{n=0}^{\infty} \frac{(-1)^k\cdot[\Gamma(\alpha+1)]^{2n}}{\Gamma(2n\alpha+1)}\ . \tag{22}$$

On the other hand, since $sin_{\alpha}(x^{\alpha}) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)\alpha}}{\Gamma((2n+1)\alpha+1)}$, it follows that

$$sin_{\alpha}(1) = \sum_{n=0}^{\infty} \frac{(-1)^n\cdot[\Gamma(\alpha+1)]^{2n+1}}{\Gamma((2n+1)\alpha+1)}\ . \tag{23}$$

Example 3.4: If $0 < \alpha \leq 1$ and r is a real number. Find $\left(\frac{3}{2}\right)^{\otimes_\alpha r}$ and $\left(\frac{5}{3}\right)^{\otimes_\alpha r}$.

Solution By fractional binomial series,

$$\left(\frac{3}{2}\right)^{\otimes_\alpha r}$$

$$= \left(1+\frac{1}{2}\right)^{\otimes_\alpha r}$$

$$= \sum_{n=0}^{\infty} \frac{(r)_n}{\Gamma(n\alpha+1)}\left[\frac{1}{2}\cdot\Gamma(\alpha+1)\right]^n$$

$$= \sum_{n=0}^{\infty} \frac{(r)_n\cdot[\Gamma(\alpha+1)]^n}{2^n\cdot\Gamma(n\alpha+1)}\ . \tag{24}$$

And

$$\left(\frac{5}{3}\right)^{\otimes_\alpha r}$$

$$= \left(1+\frac{2}{3}\right)^{\otimes_\alpha r}$$

$$= \sum_{n=0}^{\infty} \frac{(r)_n}{\Gamma(n\alpha+1)}\left[\frac{2}{3}\cdot\Gamma(\alpha+1)\right]^n$$

$$= \sum_{n=0}^{\infty} \frac{(r)_n\cdot 2^n\cdot[\Gamma(\alpha+1)]^n}{3^n\cdot\Gamma(n\alpha+1)}\ . \tag{25}$$

Example 3.5: Suppose that $0 < \alpha \leq 1$. Find T_{α} .

Solution If $\left(\frac{1}{\Gamma(\alpha+1)}x^{\alpha}\right)^{\otimes_\alpha 2} < 1$, then

$$arctan_\alpha(x^\alpha)$$

$$=\left({}_0I_x^\alpha\right)\left[\left[1+\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 2}\right]^{\otimes_\alpha -1}\right]$$

$$=\left({}_0I_x^\alpha\right)\left[1-\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 2}+\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 4}-\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 6}+\cdots\right]$$

$$=\frac{1}{\Gamma(\alpha+1)}x^\alpha-\frac{1}{3}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 3}+\frac{1}{5}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 5}-\frac{1}{7}\left(\frac{1}{\Gamma(\alpha+1)}x^\alpha\right)^{\otimes_\alpha 7}+\cdots$$

$$=\frac{1}{\Gamma(\alpha+1)}x^\alpha-\frac{2!}{\Gamma(3\alpha+1)}x^{3\alpha}+\frac{4!}{\Gamma(5\alpha+1)}x^{5\alpha}-\frac{6!}{\Gamma(7\alpha+1)}x^{7\alpha}+\cdots$$

$$=\sum_{n=0}^{\infty}(-1)^n\frac{(2n)!}{\Gamma((2n+1)\alpha+1)}x^{(2n+1)\alpha}\,. \tag{26}$$

Hence,

$$\frac{T_\alpha}{8}=arctan_\alpha(1)=\sum_{n=0}^{\infty}(-1)^n\frac{(2n)!\cdot[\Gamma(\alpha+1)]^{(2n+1)}}{\Gamma((2n+1)\alpha+1)}\,.$$

That is,

$$T_\alpha=8\cdot\sum_{n=0}^{\infty}(-1)^n\frac{(2n)!\cdot[\Gamma(\alpha+1)]^{(2n+1)}}{\Gamma((2n+1)\alpha+1)}\,, \tag{27}$$

if $\sum_{n=0}^{\infty}(-1)^n\frac{(2n)!\cdot[\Gamma(\alpha+1)]^{(2n+1)}}{\Gamma((2n+1)\alpha+1)}$ exists.

IV. CONCLUSION

In this paper, we obtain the infinite series expressions for the values of some fractional analytic functions. Jumarie type of R-L fractional calculus and a new multiplication of fractional analytic functions play important roles in this paper. In fact, our results are generalizations of traditional calculus results. In the future, we will continue to study the problems in fractional differential equations and applied mathematics.

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